HO 2: Trigonometry

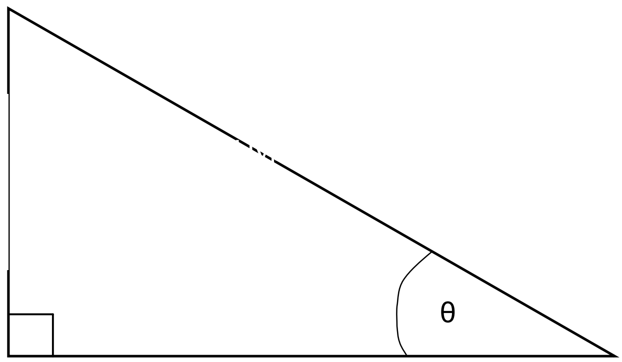
A very short history!
Trigonometry, primarily the relationship between the lengths of the sides and the angles in triangles, dates from the 2nd century BC. Both the Egyptians and Indian mathematicians had made use of their study of triangles; the former in the context of pyramids* and the later in the context of holy altars.

It was the Greeks though who fully developed the idea and it was Hipparchus who first compiled tables of trigonometric functions. His key interest was in calculating and predicting the position of planets and he used imaginary triangles in the sky to do this.

Indian and Arab mathematicians continued the study and it was Hindu mathematicians who first used sines as we recognise them today.

Modern trigonometry dates from the mid-1500s when its principles were applied to clocks, engineering, construction, artillery and navigation.

Trigonometry in right angled triangles
Trig is used to work out the lengths and angles in right angled triangles. If you are 'given', or can measure, an angle and a side it is possible to calculate the missing sides. This is still very important to engineers, surveyors, architects and astronomers today.



The hypotenuse is always the longest side and is the side opposite the right angle.

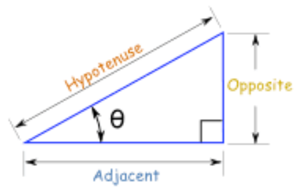
The adjacent side is the side next to the angle being considered.

The opposite side is the side opposite the angle being considered.

(NB the adjacent and opposite sides are named with respect to a given angle. If the other angle in the triangle in the diagram had been marked then the adjacent and opposite sides would be reversed.)

Trig Ratios
In right angled triangles the ratio of the lengths of sides relate to the angles within the triangle. So for the above diagram:

For any angle "**θ**":



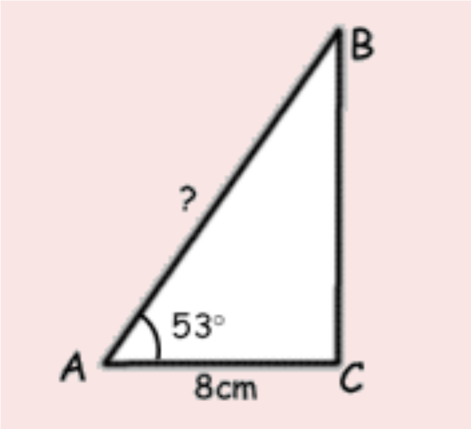
Sine Function: **sin(θ) = Opposite / Hypotenuse**

Cosine Function: **cos(θ) = Adjacent / Hypotenuse**

Tangent Function: **tan(θ) = Opposite / Adjacent**

(Sine, Cosine and Tangent are often abbreviated to sin, cos and tan.)

Worked example

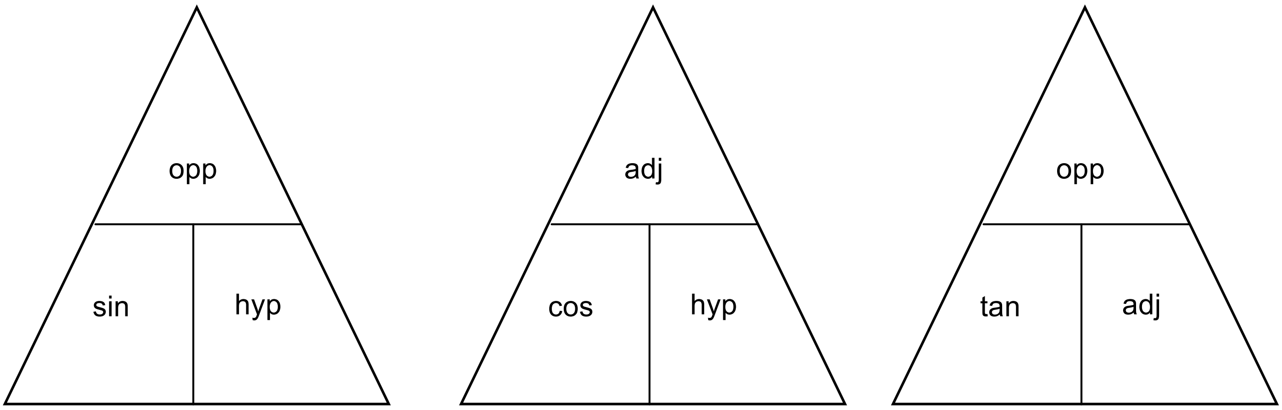


What is the length of side c, when b = 8 cm and angle A is 53°?
The first step is to identify which of the ratios we can use. We need to find the length of the **Hypotenuse** and we know the length of the **Adjacent** side. Looking at **SOHCAHTOA** we can use the cosine ratio.
So cos A = b/c
So cos 53 = 8/c
Rearrange the formula by multiplying both sides by c.
cos 53 x c = 8
Rearrange the formula by dividing both sides by cos 53.
So c = 8/cos 53

This has the effect of putting the side you are looking for on its own on one side of the equation. If you are confident about rearranging formulae you could do both the steps above at the same time. If you are not so confident, do each stage separately. It may seem to take a long time but you can check that you are right.
You will need to use your calculator to work out the cosine of 53°
To do this input 53 then press the cos key.
cos 53 = 0.6018 (4dp)
So c = 8/0.6018
Therefore c= 13.29cm (2dp)

This example is from https://www.liverpool.ac.uk/~cll/lskills/WN/Numeracytriangles.html which you can work through a step at a time.

Rearrangement of formula
The following method can be used for rearranging the three trig ratios.



From the diagrams we get:

opp = sin x hyp	adj = cos x hyp	opp = tan x adj
sin = opp/hyp	cos = adj/hyp	tan = opp/adj
hyp = opp/sin	hyp = adj/cos	adj = opp/tan

Try it out https://www.mathsisfun.com/sine-cosine-tangent.html scroll down the page and move the mouse around.