Level 3 Award in Mathematics for numeracy Teaching: Session 4: Shape and Space

Session 2's number questions

1. A company usually sends 9 people to install a security system in an office building, and they do it in about 96 minutes. Today, they have only three people to do the same job. How much time should be scheduled to complete the job?
2. A dog trainer has to feed vitamins to his adult dogs. The dosage for adult dogs weighing 20 pounds is 2 teaspoons per day. He needs to feed vitamins to a male dog weighing 75 pounds and to a female dog weighing 7 pounds. Determine the correct dosage for both male and female dogs. Note any assumptions you have made.
3. What speed covers 27 miles in 3 hours?
4. At 13mph, how far do you travel in 2 hours?
5. Write the number 0.00037 in standard form.
6. Write 6.43×10^5 as an ordinary number.
7. Work out the value of $2 \times 10^7 \times 8 \times 10^{-12}$ Give your answer in standard form.
8. Work out the value of $3 \times 10^{-5} \times 40{,}000{,}000$
9. The surface area of Earth is $510{,}072{,}000 km^2$. The surface area of Jupiter is $6.21795 \times 10^{10} km^2$. The surface area of Jupiter is greater than the surface area of Earth. How many times greater? Give you answer in standard form.

Quadratic equations

Zarig took part in a 26 mile road race.

a) He ran the first 15 miles at an average speed of x mph. He ran the last 11 miles at an average speed of (x-2) mph. Write down an expression, in terms of x, for the time he took to complete the 26 mile race

b) Zarig took 4 hours to complete the race. Using your answer to part a), form an equation in terms of x

c) (i) Simplify your equation and show that it can be written as $2x^2-17x+15=0$

(ii) Solve this equation and obtain Zarig's average speed over the first 15 miles of this race

Quadratic equations

	Average speed	Distance	Time
First 15 miles	x	15	?
Last 11 miles	x - 2	11	?

a)

Time = distance/speed

Time = $\frac{15}{x} + \frac{11}{(x-2)}$

b) 4 = $\frac{15}{x} + \frac{11}{(x-2)}$

Quadratic equations

	Average speed	Distance	Time
First 15 miles	x	15	?
Last 11 miles	x - 2	11	?

C i) $4 = \frac{15}{x} + \frac{11}{(x-2)}$

Multiply by x

$4x = 15 + \frac{11x}{(x-2)}$

Multiply by $(x - 2)$

$(4x)(x - 2) = 15(x - 2) + 11x$

$4x^2 - 8x = 15x - 30 + 11x$

$4x^2 - 8x = 26x - 30$

$4x^2 - 8x - 26x + 30 = 0$

$4x^2 - 34x + 30 = 0$

Divide by 2

$2x^2 - 17x + 15 = 0$

Quadratic equations

	Average speed	Distance	Time
First 15 miles	x	15	?
Last 11 miles	x - 2	11	?

Cii Average speed over first 15 miles

$2x^2 - 17x + 15 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

$$x = 17 \pm \frac{\sqrt{[(17 * 17) - 4 * 2 * (+15)]}}{2*2}$$

$$x = 17 \pm \frac{\sqrt{[(289) - 120]}}{4}$$

$$x = 17 \pm \frac{\sqrt{[(289) - 120]}}{4}$$

$$x = \frac{17 \pm \sqrt{169}}{4}$$

$$x = \frac{17 \pm 13}{4}$$

$$x = \frac{30}{4} = \mathbf{7.5}$$

Or

$$x = \frac{17 - 13}{4} = \mathbf{1}$$

Quadratic equations

	Average speed	Distance	Time
First 15 miles	x	15	?
Last 11 miles	x - 2	11	?

How about factorizing rather than using the quadratic equation?

$2x^2 - 17x + 15 = 0$

$(2x - 15)(x - 1) = 0$

So x can equal either 7.5 or 1 !!!

Why can't it equal 1?

Aims

To reflect upon personal maths skills and identified areas for development

To revise / widen participants' personal mathematical skills relating to geometry and trigonometry.

What do you know about geometry?

Bearing

Area

2D shapes (excluding quadrilaterals)

Perimeter

Regular and irregular shapes

Quadrilaterals (names and properties)

Internal and external angles

3D shapes

Transformations

Tessilations

Ratios of length, area and volume

Vector

Congruent shapes

Similar shapes

Circles

Triangles

Angles

Shape and space: geometry

Some examples of the concepts you may wish to investigate further:

- Tessilations
- Perimeter
- Area
- Plans and working to scale
- Transformations (rotations, transitions, reflections)
- Vectors
- Bearings

Shape and space: geometry

R2 classifying shapes

Have a play for 5 minutes.

Shape and space: geometry

R3 Dissecting a square – what do you know that you can apply?

Shape and space: geometry

R4 Transformations

You might find it useful (particularly if you are going to teach this) to look at:

https://www.youtube.com/watch?v=iApE9gOj3f4&feature=youtu.be **(from 3:30)**

https://www.khanacademy.org/math/cc-eighth-grade-math/cc-8th-geometry/translations-8th/v/introduction-to-transformations

Shape and space: geometry

I will give you some instructions – you need to be very accurate

- Take a piece of paper (if everyone uses the same size paper we can do a comparison at the end)
- Put it on the desk landscape.
- Mark with pencil anywhere they like on the top long edge.
- Using a rule draw a line from this mark to the bottom right hand corner
- Using a rule draw a second line from the first mark to the bottom left hand corner
- You should now have a triangle?
- Cut out the triangle with scissors
- Fit the two smaller pieces into the large triangle – they should fit exactly if you have done this accurately (if not you can have another go later – once you know what to do it is easier)
- What does this tell us – what can we generalise from this?

Shape and space: geometry

R6 – is the cut up A4 sheet useful when deciding whether the statement is always, sometimes or never true?

What do you know about trigonometry?

Trigonometry is all about triangles.

Vocabulary:

- 3 sides, 3 angles
- Adjacent
- Opposite
- Hypotenuse
- Sine, Cosine, Tangent – are functions

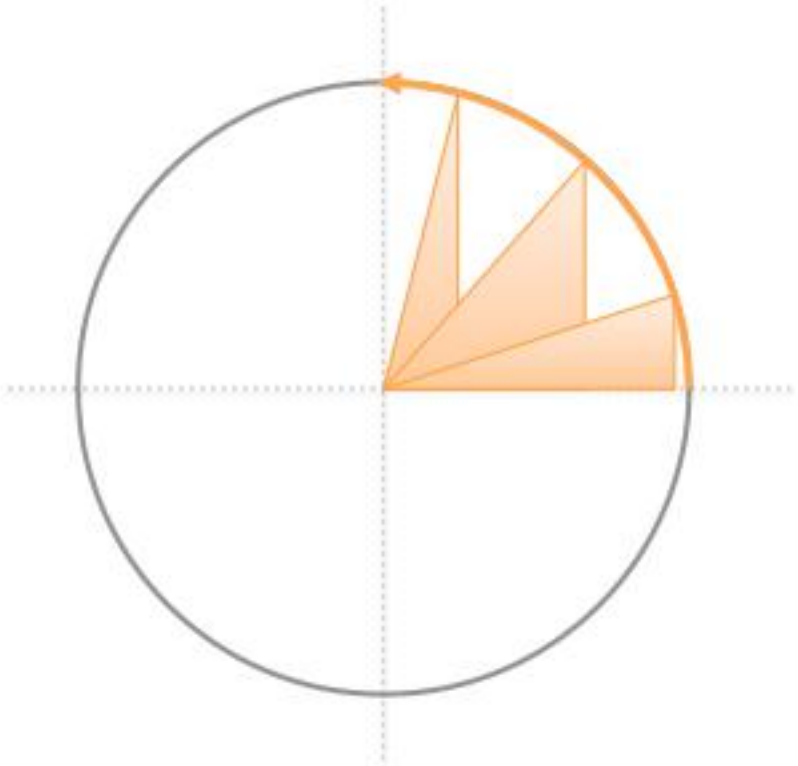
- The handout on trigonometry on the website outlines some of these.

What do you know about trigonometry?

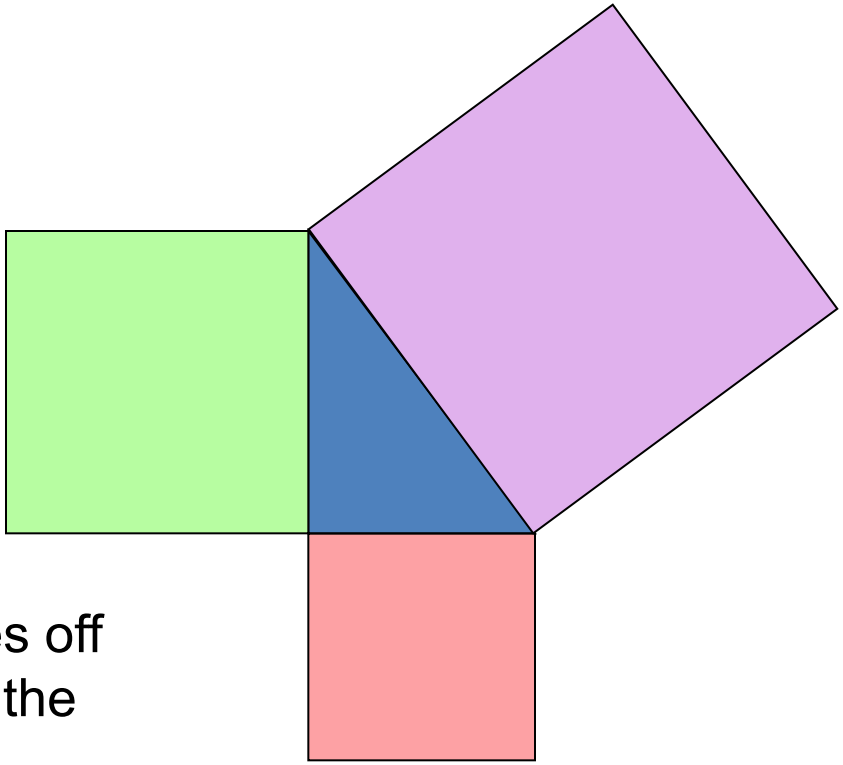
Three theorems shared by geometry and trigonometry

- The sum of the internal angles of any triangle is equal to two right angles
- Pythagorean theorem
- Similar triangles

Triangles show up in circles!



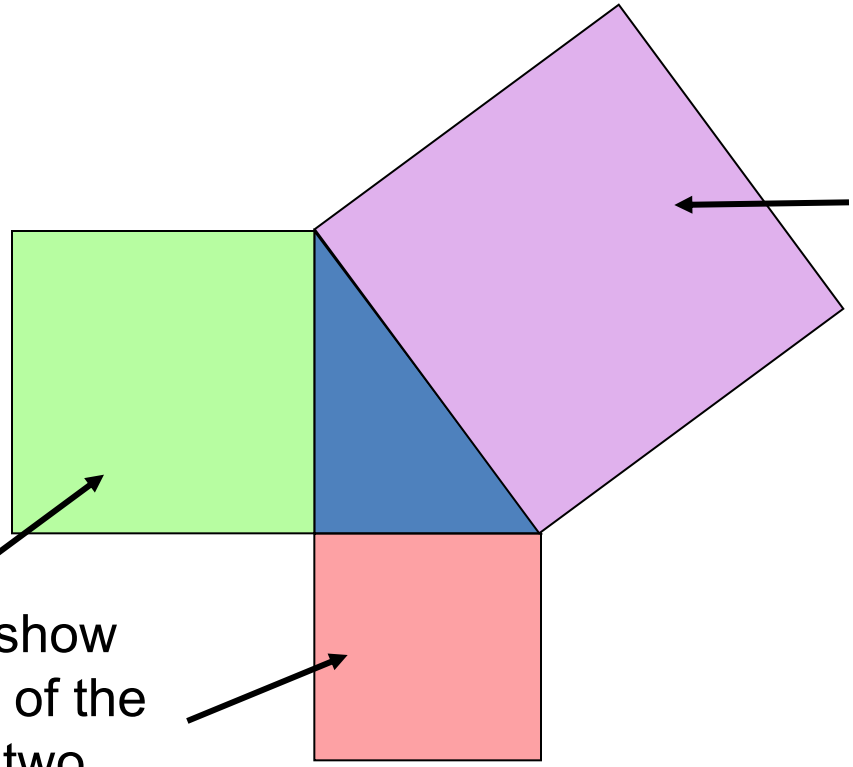
Pythagoras' theorem



Take a right-angled triangle with sides of 5cm, 4cm and 3cm.

Draw squares off each side of the triangle.

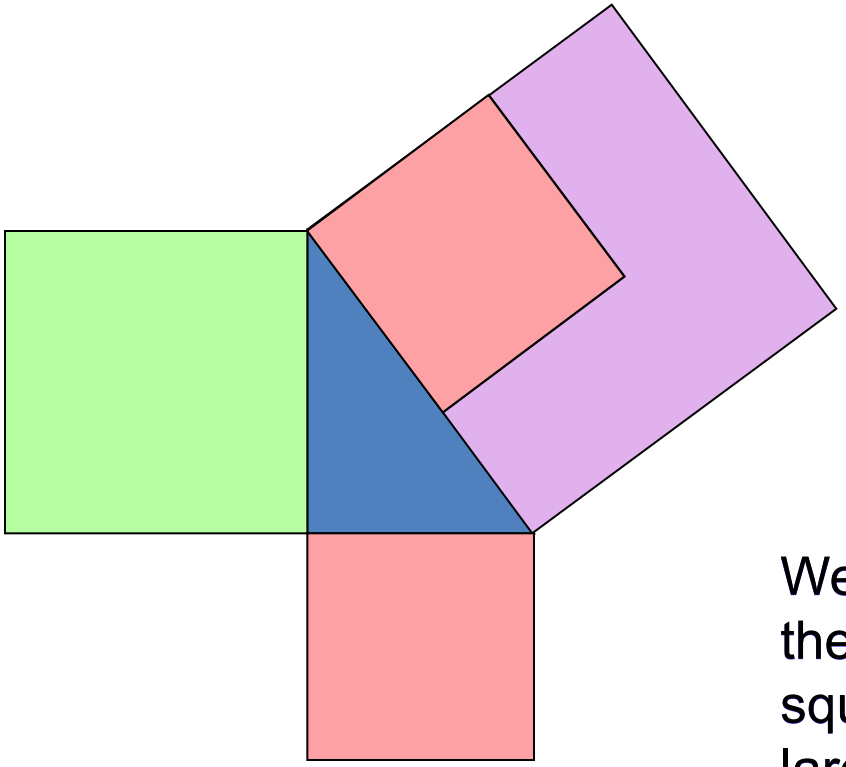
Pythagoras' theorem



… is equal to the area of the larger square.

We need to show that the sum of the areas of the two smaller squares…

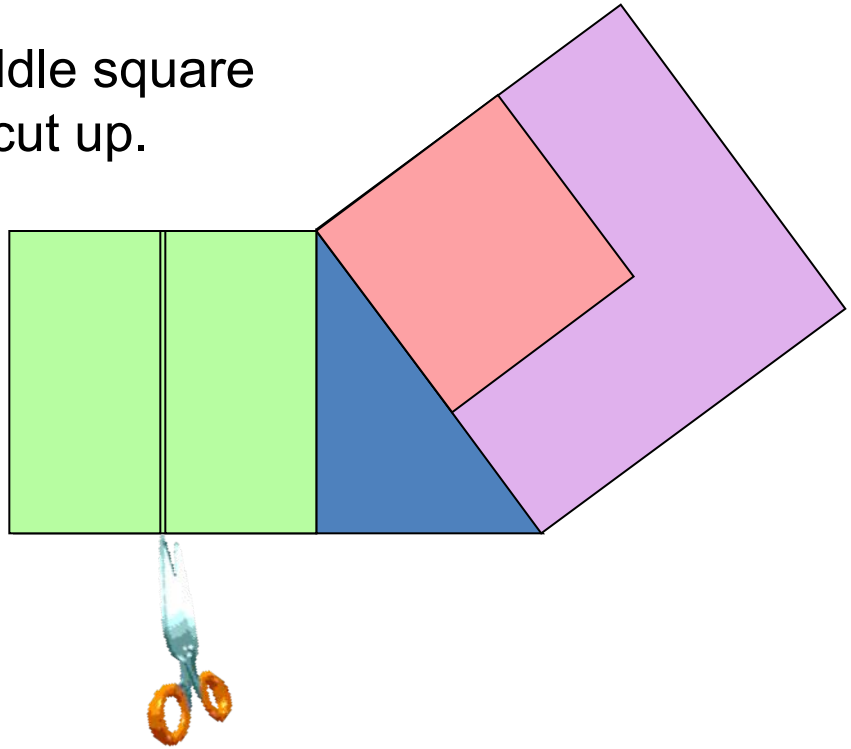
Pythagoras' theorem



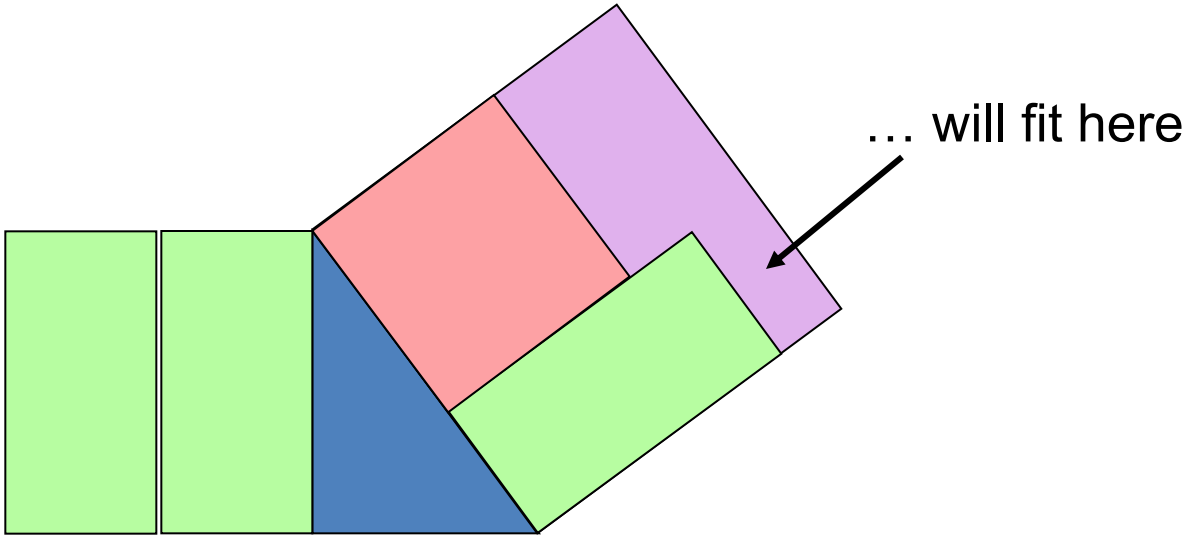
We can move the smallest square onto the largest square.

Pythagoras' theorem

The middle square can be cut up.

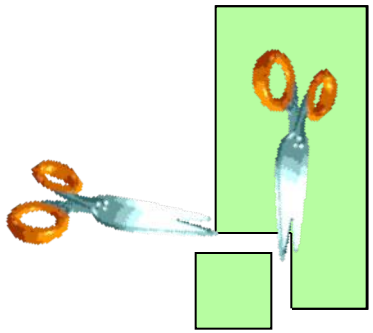


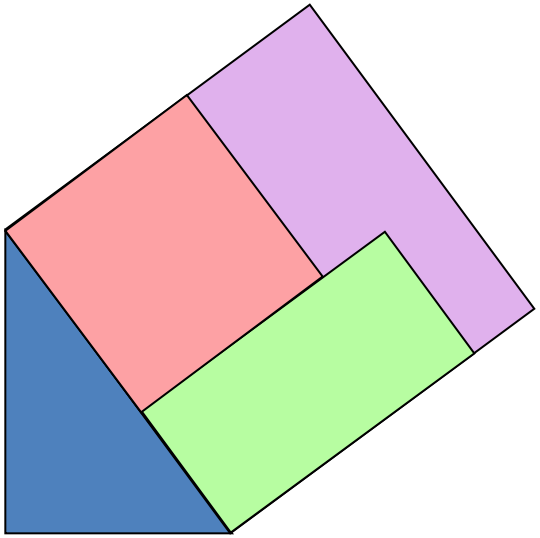
Pythagoras' theorem



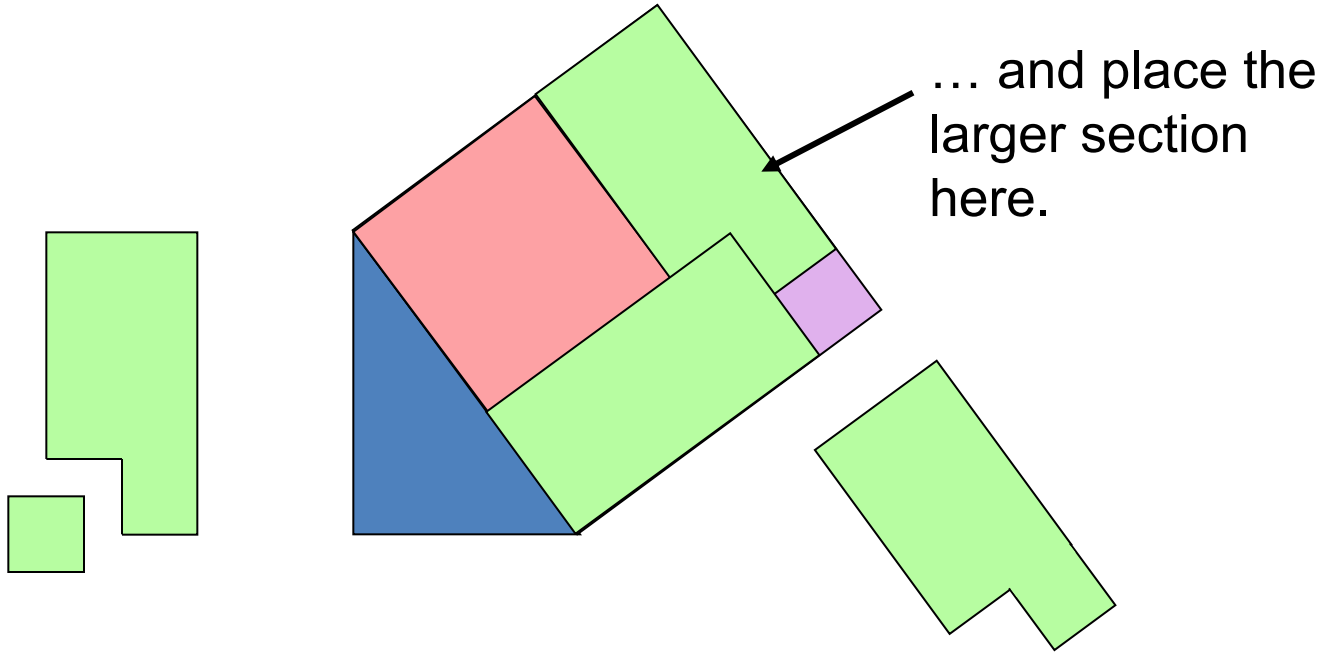
Pythagoras' theorem

We can cut out a square from this section…



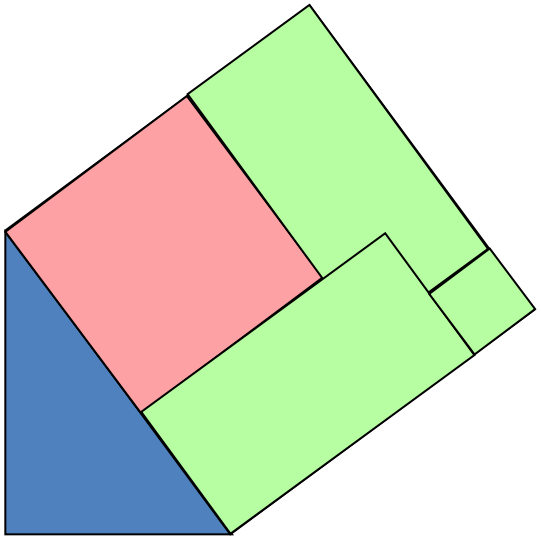


Pythagoras' theorem



Pythagoras' Theorem

The small square…

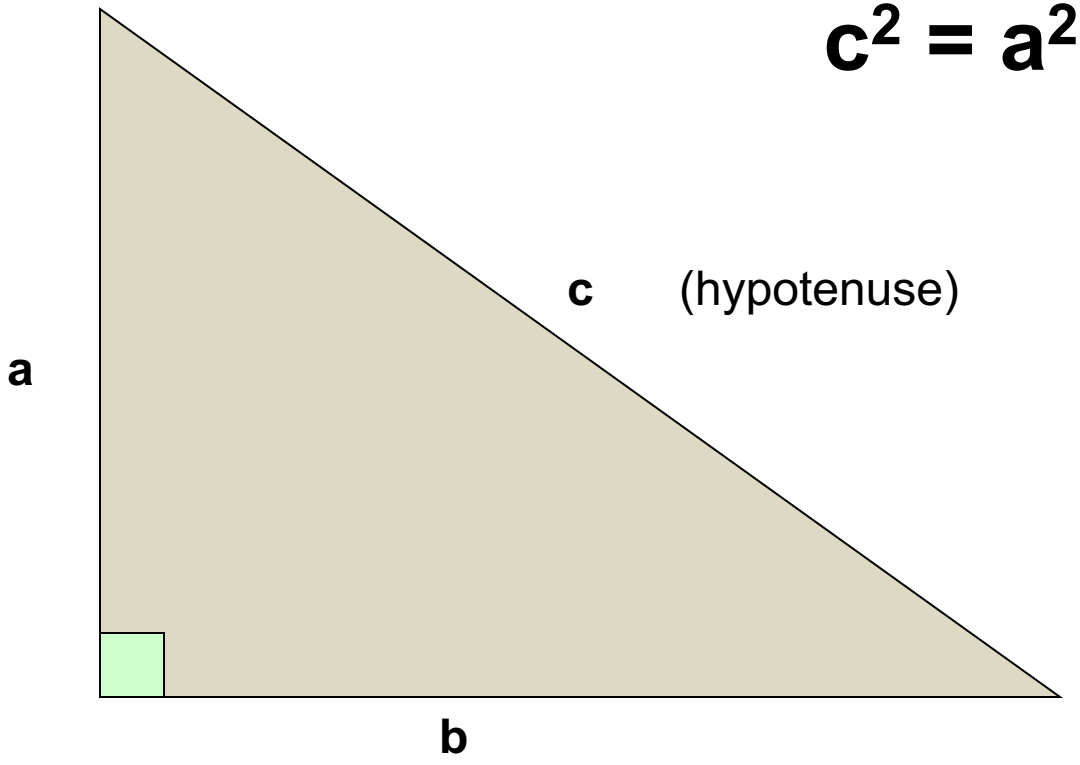


…will fit here

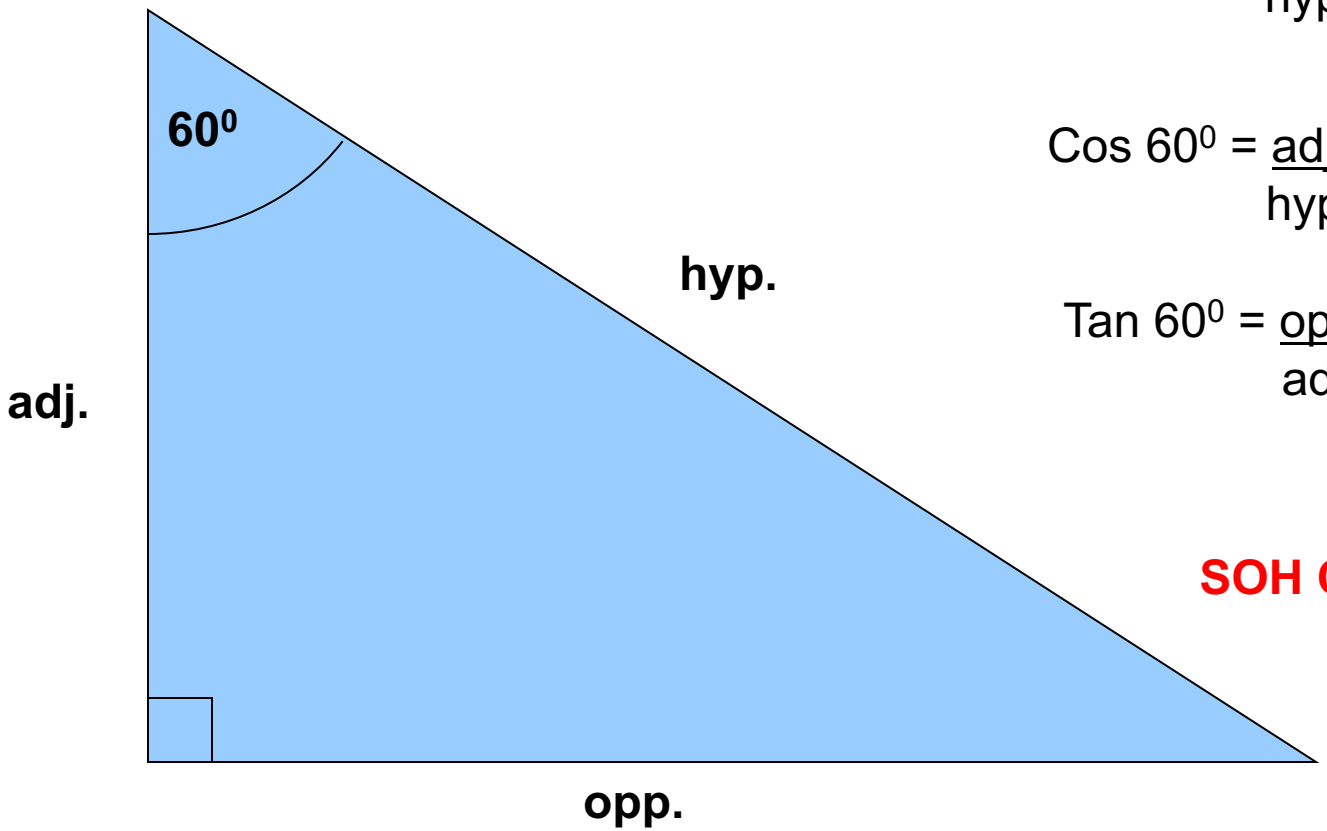
Would this work with any right-angled triangle?

Pythagoras' theorem

$$c^2 = a^2 + b^2$$



Trig ratios



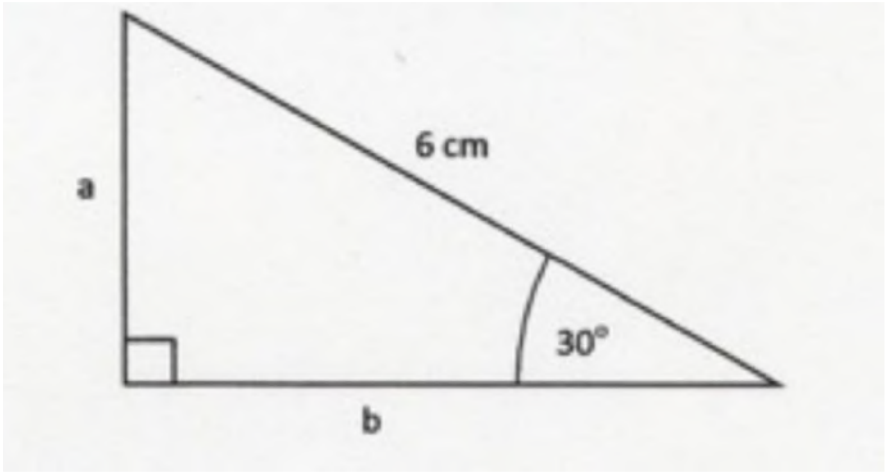
$\text{Sin } 60^0 = \frac{\text{opp}}{\text{hyp}}$

$\text{Cos } 60^0 = \frac{\text{adj}}{\text{hyp}}$

$\text{Tan } 60^0 = \frac{\text{opp}}{\text{adj}}$

SOH CAH TOA

R9 Trig practise questions



What to do first?

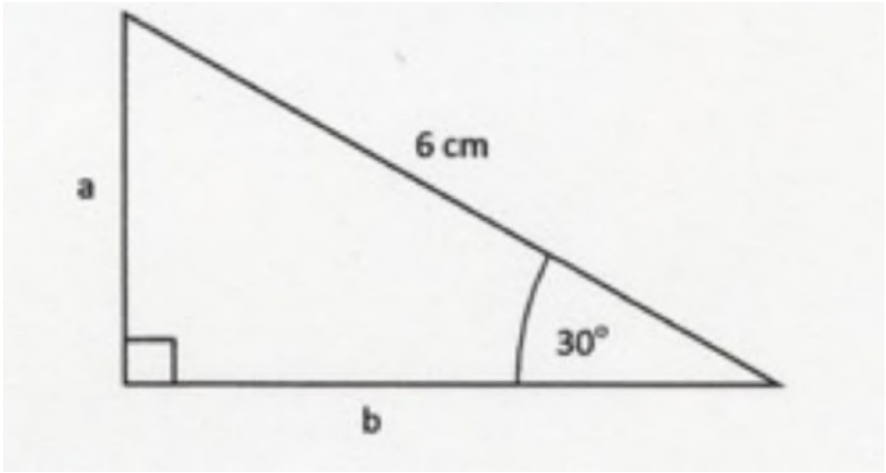
Which of the trig ratios should we use? Why?

$\sin 30^0 = \frac{\text{opp}}{\text{hyp}}$

$\cos 30^0 = \frac{\text{adj}}{\text{hyp}}$

$\tan 30^0 = \frac{\text{opp}}{\text{adj}}$

R9 Trig practice questions



What to do first?

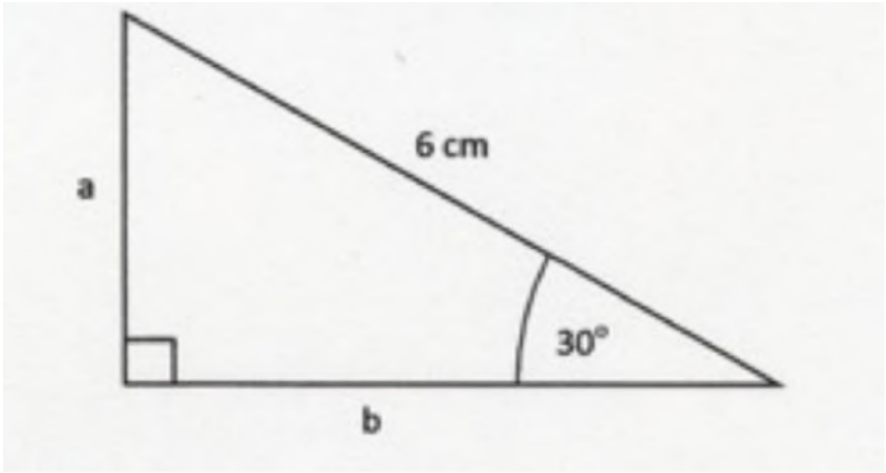
Which of the trig ratios should we use? Why?

$\sin 30^0 = \frac{\text{opp}}{\text{hyp}} = \frac{a}{6}$

$\cos 30^0 = \frac{\text{adj}}{\text{hyp}} = \frac{b}{6}$

$\tan 30^0 = \frac{\text{opp}}{\text{adj}} = \frac{a}{b}$

R9 Trig practice questions



$\text{Sin } 30^0 = \frac{\text{opp}}{\text{hyp}} = \frac{a}{6}$

$\text{Sin } 30^0 \times 6 = a$

$a = \text{Sin } 30^0 \times 6$

$a = 0.5 \times 6 = 3$

$\text{Cos } 30^0 = \frac{\text{adj}}{\text{hyp}} = \frac{b}{6}$

$\text{Cos } 30^0 \times 6 = b$

$b = \text{Cos } 30^0 \times 6$

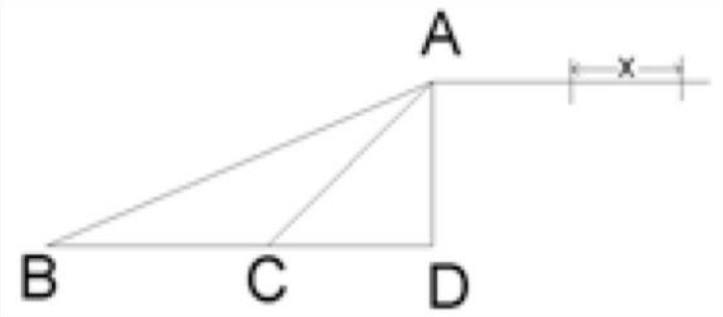
$b = 0.8660\text{'sh} \times 6$

$b = 5.1961$ or 5.2'sh

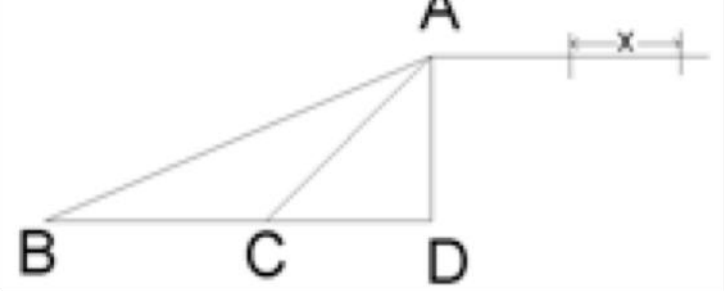
Let's have a play with a question

- Imagine you are on a jetty, and you are pulling in a boat that is floating on the water some way away. The rope comes up over the edge of the jetty, and lies along the jetty as you pull the boat in.
- As you pull in 10 metres of rope, the boat clearly moves in too, but does it move *exactly* 10 metres, *more than* 10 metres or *less than* 10 metres?
- **So let me know in chat – is it exactly, more, or less than 10m?**
- **Why not draw a sketch – it might help (and think back to some of the algebra we have done).**

Let’s have a play with a question



Let's have a play with a question



As the rope is pulled a distance x, the boat moves from, say, B to C.
So CA = BA - x

For any triangle, the sum of any two sides is greater than the third, so
BC + CA > BA

Hence, BC + (BA - x) > BA

Hence, BC - x > 0, or, equivalently, BC > x

So the boat comes in <u>further</u> than you pull the rope.

Nice, isn't it?

Pythagoras' theorem

Useful links

https://en.wikipedia.org/wiki/Pythagorean_theorem

http://www.mathsisfun.com/pythagoras.html

https://www.khanacademy.org/math/trigonometry/trigonometry-right-triangles/intro-to-the-trig-ratios/a/finding-trig-ratios-in-right-triangles

https://www.youtube.com/watch?v=VRz2d5yedsg

A sample question for you to have a go at:
- Areas and volumes
- Scale drawings and trigonometry

In week 1 we looked at a practical application of shape and space

What other mathematical skills might we need when working on this activity?

Wk1, R14 Packing chocolates - task

A chocolate manufacturer produces bars shaped in a triangular prism of length 12cm. The cross section of the bars is an equilateral triangle of side length 4cm.

Design a net for the packaging of the individual bars.

The packaged bars are to be put into boxes that are 40 x 40 x 40 cm^3.

Find the maximum number of bars that can be packed into each box.

1. Draw a diagram of the package for the chocolate. Show all the measurements for the dimensions of the package.
2. Design a net to produce the package. Take care to add flaps that will be needed to stick edges together.
3. To fit the packages in the box, think about layers. How many packages would cover the base of the box? Being triangular prisms, some packages could go upside down to fit neatly on the packages that are covering the base. All of these make up one layer. How many packages in one layer?
4. Work out the height of a layer. How many layers fit in the box?
5. Is there any spare space for more packages to fit in? Consider the layers and the spare space and calculate the total number of packages that can fit in a 40 x 40 x 40 cm^3 box.

Areas and Volumes :

Workmen are digging a ditch which is to be 30m long and 1.25m deep. The cross-section is a symmetrical trapezium. The ditch must be 1.5m wide at the bottom and 2.5m wide at the top.

- Draw and label a diagram of the cross-section
- Find the area of the cross-section
- Calculate how many cubic metres of dirt are to be excavated from the ditch

The workmen use an excavator and the dirt is taken away in lorries that can take $12m^3$ at a time

- How many full loads of dirt are taken away?
- The last lorry is only partially filled up. What fraction of a full load does the last lorry take?

Scale Drawings and Trigonometry:

A car travels from its base on a bearing of 030° for 12 km. It then turns and drives a further 8 km on a bearing of 100° after which it breaks down.

- a) Draw a clear sketch of this journey, stating all the known lengths and angles
- b) A recovery vehicle leaves base to pick up the car. How far must it travel and on what bearing should it head?

Hint – draw a sketch and remind yourself about bearings at https://corbettmaths.com/tag/bearings/